BENDING RESONANT WAVES IN A SHELL WITH ASSOCIATED MASSES

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In publications devoted to studying nonstationary deformation of extended cylindrical structures under the action of moving surface loads (see e.g. [1-3]) considerable attention is given to analyzing dispersion curves (dependence of phase velocity c in wave number q) which make it possible to reveal singular points (q_*, c_*) where phase and group ($c_g = c + q\,dc/dq$) velocities are equal. It was shown in [2] that group waves with lengths close to $2\pi/q_*$ propagate almost without dispersion and they form a wave packet with a quasistationary envelope moving with constant velocity c_*. The value $c = c_*$ determines the critical velocity of load movement. Numerical modelling of bending resonance waves in a cylindrical shell is given in [4], and for a more complex system, i.e., for a shell with a damping medium, it is given in [5] where it is also shown that the concentrated part of the spectrum in relation to parameters of the system may be a different number of singular points, and in spite of the same level in the growth of resonance waves of a specific asymptotic the qualitative picture of the development of perturbations with different c_{j*} (j = 1, ..., m; m is number of singular points) is different. This fact, detected in a numerical experiment, required theoretical substantiation and additional analysis of the results which is given below.

A pure bending stressed state is considered for an infinitely long thin cylindrical shell in contact with a medium having a density of m_0 and a damping frequency of f^2. The set of equations of motion in modelling the medium by inertial masses has the form

$$\ddot{w} = -\varepsilon w_{,x}^{(\mathrm{IV})} - w + h^{-1}\overline{m}_0 f^2 (W - w) + Q_1 h^{-1},$$
$$\ddot{W} = f^2 (w - W) + Q_2 m_0^{-1}, \quad \varepsilon = h^2/12, \quad \overline{m}_0 = m_0/\rho, \tag{1.1}$$

where w, W are displacements of the shell and damping masses $Q_{1,2} = A_{1,2} H_0(t) H_0(c_0 t - |x|)$ are loads applied to the shell and masses ($A_{1,2}$ are amplitudes, $H_0(z)$ is Heavyside function); h is shell thickness; sound velocity in a thin plate $c_p = \sqrt{E/[\rho(1-\nu^2)]}$, shell material density ρ, and its radius R serve as unit of measurement.

As a result of loading symmetry it is sufficient to consider region $x \geq 0$ when in plane x = 0 boundary conditions $w'_{,x} = w'''_{,x} = 0$ $(x = 0)$ should be fulfilled. The initial conditions are zero conditions, and with $x \to \infty$ the following conditions is set for the study $w, W \to 0$ $(x \to \infty)$.

We apply to (1.1) integral Laplace transforms with respect to t (with parameter p) and Fourier transforms with respect to x (with parameter q). We give the two-stage transform the symbol $()^{LF}$. Then the solution in the transforms is written as:

$$w^{LF} = \frac{(p^2 + f^2) Q_1^{LF} + f^2 Q_2^{LF}}{hA(p, q)},$$
$$W^{LF} = \frac{(p^2 + \varepsilon q^4 + 1) h m_0^{-1} Q_2^{LF} + f^2 \left(Q_1^{LF} + Q_2^{LF}\right)}{hA(p, q)}, \tag{1.2}$$

where A(p, q) is dispersion operator:

$$A(p, q) = (p^2 + \varepsilon q^4 + 1)(p^2 + f^2) + h^{-1}\overline{m}_0 f^2 p^2. \tag{1.3}$$

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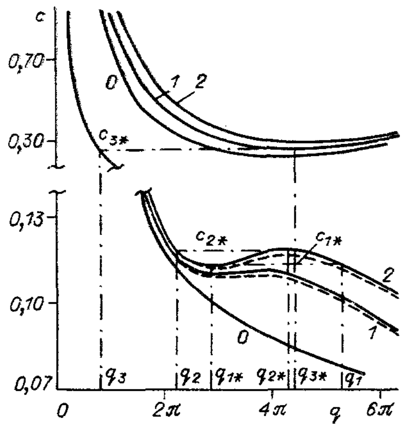

Fig. 1

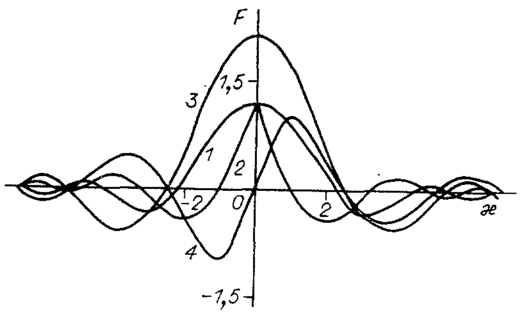

Fig. 2

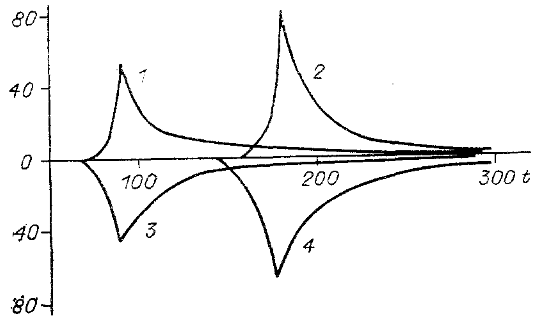

Fig. 3

Now we consider the dispersion properties of harmonic waves which propagate in the test system. In (1.3) we place p = iqc (q is wave number, c is phase velocity) and we equate it to zero. As a result of this we obtain a second-degree equation with respect to c^2 which determines the two modes of harmonic vibrations. Its roots have the form

$$c_1(q) = \sqrt{(a-b)/2},\ c_2(q) = \sqrt{(a+b)/2},\ a = dq^{-2} + \varepsilon q^2,$$
$$b = \sqrt{a^2 - 4f^2(1+\varepsilon q^4)q^{-4}},\ d = f^2(1 + h^{-1}\bar{m}_0) + 1. \tag{1.4}$$

Here depending on shell stiffness, density of the medium, and damping frequency, appearance is possible on the dispersion curve for the first (lower) mode of different extremes (c_{j*}, j = 1, 2), a minimum, a maximum, and a point of inflection, and the second mode always has a minimum c_{3*} (solid curves in Fig. 1; numbers 0, 1, 2 correspond to f^2 = 1; 3.6; 4.9, m_0 = 0.05), which have a contact with phase curves at points $q = q_{j*}$, whereas they intersect the curve for the first mode at points q_j (j = 1, 2, 3). Thus, with movement of a load with critical velocity c perturbations form simultaneously in the system with wavelengths $2\pi/q_{j*}$ and $2\pi/q_j$, and the contribution of resonant waves with a frequency of the form q_{j*}, in the overall process is previously unknown in a finite time interval. How the proportion of energy takes one or another mode depends on the ratio of amplitudes in breaking down the load with respect to forms of movements which relate to this mode, and on the magnitude of group velocities at points $q = q_j$.

We turn to solving set (1.1). It is a problem to obtain the originals in the explicit form of (1.2), and therefore we shall find the asymptotics of perturbations with long times ($t \to \infty$) from the start of load operation. For this we use the method of combined treatment of two-stage integral transforms in the vicinity of ray $x = c_{j*}t + \eta$, $\eta = \text{const}$ [2]. In changing over to a ray in transforms f^{LF} (p, q) of the functions sought a substitution is made $p = s + i\bar{q}_j c_{j*} + iq'x/t$ ($s \to 0$, $\bar{q}_j$ takes the value q_{j*} or q_j, and q' is a small value determining the vicinity of points $q = \pm\bar{q}_j$ for which subsequent integration is carried out).

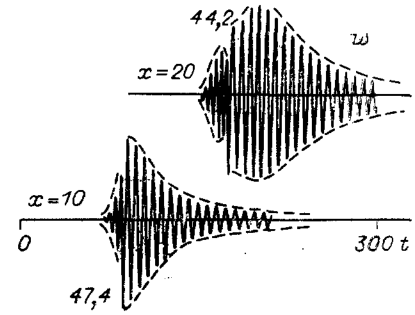

Fig. 4

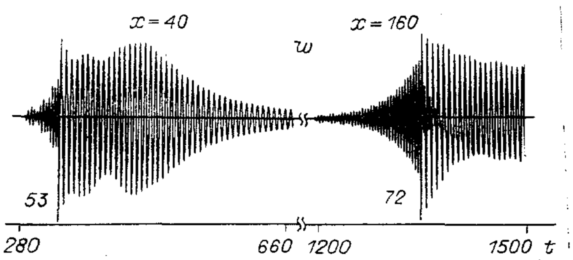

Fig. 5

By omitting intermediate calculations we write the asymptotic equations obtained for evaluation of perturbations (retaining the contribution from nonincreasing components of the solution). When the velocity of the load $c = c_{j*}$ ($j = 1, 2$) corresponds to the maximum (minimum) the dispersion curve for the first mode of the asymptotic ($t \to \infty$) of perturbations has the form

$$
\begin{gathered}
w, W \sim B_{w,W}\left\{\left(\operatorname{sgn}\eta + \left[C\left(\frac{\varkappa_0^2}{4}\right) + S\left(\frac{\varkappa_0^2}{4}\right)\right]\operatorname{sgn}\varkappa_0\right)\cos(q_j\eta) + \right.\\
\left. + \left[C\left(\frac{\varkappa_0^2}{4}\right) - S\left(\frac{\varkappa_0^2}{4}\right)\right]\operatorname{sgn}\varkappa_0 \operatorname{sgn}\varphi_0 \sin(q_j\eta)\right\} + \\
+ D_{w,W}t^{1/2}\left[F_1(\varkappa)\cos(q_{j*}\eta) + F_2(\varkappa)\sin(q_{j*}\eta)\operatorname{sgn}\varphi\right],\\
B_w = -\frac{(A_1 + A_2)f^2 - A_1 q_j^2 c_{j*}^2}{\psi}, \quad D_w = -\frac{(A_1 + A_2)f^2 - A_1 q_{j*}^2 c_{j*}^2}{\chi},\\
B_W = -\frac{(A_1 + A_2)f^2 + A_2\left(\varepsilon q_j^4 + 1 - q_j^2 c_{j*}^2\right)hm_0^{-1}}{\psi},\\
D_W = -\frac{(A_1 + A_2)f^2 + A_2\left(\varepsilon q_{j*}^4 + 1 - q_{j*}^2 c_{j*}^2\right)hm_0^{-1}}{\chi},\\
\eta = x - c_{j*}t, \quad \varkappa_0 = \left(c_{g1}^0 t - x\right)\left(|\varphi_0|t\right)^{-1/2}, \quad \varkappa = -\eta\left(|\varphi|t\right)^{-1/2},\\
\varphi_0 = \frac{1}{2}\frac{dc_{g1}}{dq}\bigg|_{q=q_j}, \quad \varphi = \frac{1}{2}\frac{dc_{g1}}{dq}\bigg|_{q=q_{j*}}, \quad c_{g1}^0 = c_{g1}(q_j),\\
c_{g1} = c_{g1}(q) = [qc_1(q)]_q', \quad \psi = 2hq_j^4 c_{j*}\left(c_2^2(q_j) - c_{j*}\right)\left(c_{g1}^0 - c_{j*}\right),\\
\chi = \pi h q_{j*}^4 c_{j*}\left(c_2^2(q_{j*}) - c_{j*}^2\right)|\varphi|^{1/2},\\
F_1(\varkappa) = \sqrt{\frac{\pi}{2}}\left(\cos\left(\frac{\varkappa^2}{4}\right) + \sin\left(\frac{\varkappa^2}{4}\right)\right) - \frac{\pi|\varkappa|}{2}\left[C\left(\frac{\varkappa^2}{4}\right) - S\left(\frac{\varkappa^2}{4}\right)\right],\\
F_2(\varkappa) = \sqrt{\frac{\pi}{2}}\left(\cos\left(\frac{\varkappa^2}{4}\right) + \sin\left(\frac{\varkappa^2}{4}\right)\right) - \frac{\pi|\varkappa|}{2}\left[1 - C\left(\frac{\varkappa^2}{4}\right) - S\left(\frac{\varkappa^2}{4}\right)\right].
\end{gathered}
\tag{1.5}
$$

Curves $F_1(\varkappa)$ and $F_2(\varkappa)$ are shown in Fig. 2 (curves 1 and 2); $S(z)$, $C(z)$ are Fresnel integrals.

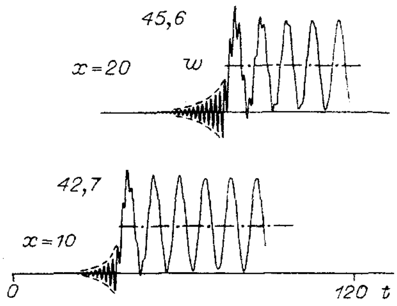

Fig. 6

Expressions similar to (1.5) may also be obtained in the case of conformity of the load velocity with the minimum of the curve for the second mode (j = 3). It can be seen from (1.5) that the second term with a common factor $D_{w,W}t^{1/2}$ grows without limit with an increase in t and with large t it determines the final values of w and W since the expression in braces standing in the factor with $B_{w,W}$ is a bounded function. For finite values of t the contribution of each of the two component terms of the asymptotic to the overall amplitude may be evaluated by comparing $B_{w,W}$ and $D_{w,W}t^{1/2}$. It is also evident that with certain values of some parameters (e.g. A_1, A_2) situations are possible when wave resonance does not form for a long time ($B_{w,W} > D_{w,W}t^{1/2}$), and the main proportion of the energy is transferred to waves with length $2\pi/q_j$.

If the critical velocity of the load corresponds to the value c_{1*}, determined by the point of inflection of the lower phase curve, then the asymptotic ($t \to \infty$) solution is written as follows:

$$
\begin{gathered}
w, W \sim R_{w,W}t^{2/3}\left(F_3(\varkappa_1) + F_4(\varkappa_1)\operatorname{sgn}\varphi_1\right)\cos(q_{1*}\eta),\\
R_w = -\frac{(A_1 + A_2)f^2 - A_1 q_{1*}^2 c_{1*}^2}{\zeta}, \quad \varkappa_1 = \eta(|\varphi_1|t)^{-1/3},\\
R_W = -\frac{(A_1 + A_2)f^2 + A_2\left(\varepsilon q_{1*}^4 + 1 - q_{1*}^2 c_{1*}^2\right)hm_0^{-1}}{\zeta},\\
\zeta = \pi h q_{1*}^4 c_{1*}\left(c_2^2(q_{1*}) - c_{1*}^2\right)|\varphi_1|^{1/3}, \quad \varphi_1 = \frac{1}{6}\left.\frac{d^2 c_{g1}}{dq^2}\right|_{q=q_{1*}},\\
F_3(\varkappa) = \int_0^\infty \frac{\cos\varkappa y \sin y^3}{y^3}\,dy, \quad F_4(\varkappa) = \int_0^\infty \frac{\sin\varkappa y\,(1 - \cos y^3)}{y^3}\,dy.
\end{gathered} \tag{1.6}
$$

Curves for functions $2F_3(\varkappa)$, $2F_4(\varkappa)$ are presented in Fig. 2 (curves 3 and 4).

By analyzing (1.6) it is easy to establish that through the system together with the load its excited wave moves with the same phase (and group) velocity and increasing amplitude in proportion to $t^{2/3}$. The degree of resonant growth compared with (1.5) is markedly higher since the order of curvature of the dispersion curve in the vicinity of the point of inflection is less than in the vicinity of other extremes.

A numerical method was used for studying the original Eqs. (1.1) in addition to the analytical method. An explicit finite-difference scheme of the "cross" type was used. Numerical dispersion was minimized by selecting optimum parameters of the network with which there is fulfillment of conditions for stability of the calculation, and the phase curves for difference and continuum in the vicinity of extremes are as close as possible. As comparison of the phase relationships for these models shows, it is only possible to obtain conformity of critical points (q_{j*}, c_{j*}) with τ, $\delta \to 0$ (τ, δ are steps of the network with respect to time and coordinate), and therefore in numerical calculations the value of c_{j*} is taken from the differential dispersion relationship, i.e., broken lines in Fig. 1 ($\tau = \delta = h = m_0 = 0.05$). In any case the second mode has a minimum (e.g. for $f^2 = 3.6$ $q_{3*} = 14.45$, $c_{3*} = 0.2610$), but the first mode with $f^2 < 3.6$ does not have any singular points, with $f^2 = 3.6$ it has a point of inflection ($q_{1*} = 10.98$, $c_{1*} = 0.1100$) and with $f^2 > 3.6$ it has two extreme points: a minimum and maximum (with $f^2 = 4.9$, $q_{1*} = 9.25$, $c_{1*} = 0.1130$, and $q_{2*} = 13.45$, $c_{2*} = 0.1163$).

Oscillograms are presented in Figs. 3-6 for deflection w of a shell calculated with $\tau = \delta = h = 0.05$ and different values of velocity c_0 for a moving stepwise load. Static deflection w in Fig. 6 is shown by a broken-dotted line (it is not considered in the rest of the cases), and the broken line relates to the quasistationary envelope. The peak amplitude is shown in the vicinity of the maximum for each curve provided.

Analysis of the results points to the fact that the maximum resonant growth ($\sim t^{2/3}$) is realized in accordance with asymptotic (1.6) in the case of $c_0 = c_{1*} = 0.1100$ ($f^2 = 3.6$), and spreading of the packet with supporting frequency $q_{1*} = 10.98$ is at a minimum and $\sim t^{1/3}$ (see Fig. 3; envelopes of oscillograms for w in sections x = 10; 20 (curves 1 and 2), $t \leq 300$, $A_1 = 1$, $A_2 = 0$). When $c_0 = c_{1*} = 0.1130$ wave resonance also arises in the system, but here bending perturbations grow with time as $t^{1/2}$, and the packet with a frequency of form $q_{1*} = 9.25$ also spreads in proportion to $t^{1/2}$ (curves 3 and 4 with the conditions indicated above).

In addition, for the same calculated values of time and $c_0 = c_{2*} = 0.1163$ the resonance process does not manage to develop (see Fig. 4: oscillograms of w in sections x = 10; 20, $t \leq 300$, $A_1 = 1$, $A_2 = 0$, static deflection is not considered). Here the maximum spreading of the wave packet is recorded in which the contribution of the "resonance" frequency of the form ($q_{2*} = 13.45$) is small, but then a wave with frequency q_2 predominates whose group velocity is much less. In comparing the numerical and analytical results acceptable qualitative and quantitative conformity is detected which apparently would not be observed with absence in (1.5) of the nonincreasing term which gives a very marked contribution in the initial stage. The delay in developing resonance in the test situation may be quite prolonged. This is confirmed in particular by oscillograms for w (see Fig. 5) obtained using asymptotic solution (1.5) in sections x = 40; 160 ($t \leq 1500$, $A_1 = 1$, $A_2 = 0$), from which it can be seen that bending perturbations with "resonant" frequency of the form $q_{2*} = 13.45$ (leading group) start to develop actually with relative large values of time (or at a considerable distance from x = 0).

If with $c_0 = 0.1163$ we consider simultaneous loading of a shell and the damping masses, and here we select values of amplitudes A_1 and A_2 so that in asymptotic (1.5) factor B_w equals zero, then it is possible to minimize the effect of perturbations with a "nonresonant" frequency of form q_j on forming the resultant wave packet and the development of resonance in the system.

The process of resonance "delay" is also realized with $c_0 = c_{3*} = 0.2610$ (see Fig. 6: oscillograms for w in sections x = 10; 20, $t \leq 300$, $A_1 = 1$, $A_2 = 0$). Here there is mainly excitation of the first mode and perturbations relating to it are described by a simple asymptotic: $w = w^0(1 - \cos q_3 c_{3*} t)$, which corresponds to the limiting case $c_0 \to \infty$.

By generalizing the results provided it is noted that in a mechanical system for which dispersion analysis reveals existence of some wave numbers q_{j*} and q_j with one and the same phase velocity c_{j*} of perturbation propagation, a long delay is possible in forming wave resonance excited by a load moving with critical velocity $c_0 = c_{j*}$. Thus, in constructing asymptotic estimates ($t \to \infty$) for perturbations which propagate in the system it is desirable to retain also "nonincreasing" terms since only in this case is it possible to obtain adquate description of the wave process in question in a finite time interval which is required in practical problems.

LITERATURE CITED

1. L. I. Slepyan, "Resonant phenomena in plates and shells with a travelling load," Proc. 6th All-Union Conf. on Shell and Plate Theory, Nauka, Moscow (1966).
2. L. I. Slepyan, Nonstationary Elastic Waves [in Russian], Sudostroenie, Leningrad (1972).
3. N. I. Pinchukova and M. V. Stepanchenko, "Low-frequency resonant waves in a cylindrical shell with associated masses," Fiz.-Tekh. Probl. Razrab. Polez. Iskop., No. 4 (1979).
4. N. I. Aleksandrova, I. A. Potashnikov, and M. V. Stepanchenko, "Bending resonant waves in a cylindrical shell with a moving load," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1989).
5. T. K. Kadyrov, I. A. Potashnikov, and M. V. Stepanchenko, "Resonant reaction of plates and shells with a medium with a travelling load," Proc. 14th All-Union Conf. on Shell and Plate Theory, Vol. 2, Tbilisi (1987).